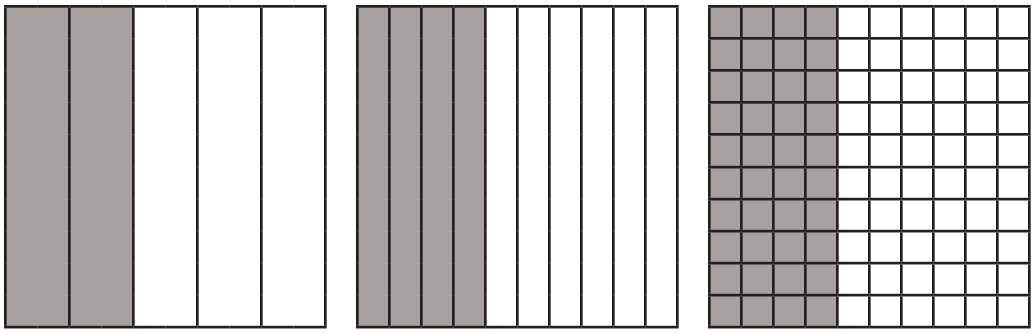
Lesson 1

Percent

We'll begin this lesson with a review of what a percentage is and then study how to find a percentage. A percentage is a fraction with a denominator of 100. It is usually easier to change a fraction to a decimal before changing it to a percentage. We can change a fraction to a decimal by placing a tenth overlay on top of it. In this illustration $\frac{2}{5}$ becomes $\frac{4}{10}$ which can be written as the decimal 0.4. We can go a step further and place the other tenth overlay on top of the first one at a 90-degree angle. Four-tenths or 0.4 is now 40/100 or 0.40 or 40%.

Figure 1



$\frac{2}{5}$ = $\frac{4}{10}$ = $\frac{40}{100}$

0.4 = 0.40

40%

Fractions as Decimals and Percentages

Every fraction can be transformed to a decimal by dividing the numerator by the denominator. The fraction $\frac{1}{2}$ is 1 divided by 2, which is 0.5. The fraction $\frac{2}{7}$ is 2 divided by 7, which is 0.285. This can be rounded to the hundredths place and becomes 0.29.

Simple fractions such as $\frac{1}{2}$, $\frac{2}{5}$, $\frac{1}{4}$, and $\frac{3}{4}$ are relatively simple to rewrite as tenths and hundredths. For example, $\frac{1}{2} = \frac{5}{10}$ and $\frac{2}{5} = \frac{4}{10}$. Think of money as you do fourths: one quarter is the same as twenty-five pennies, so $\frac{1}{4} = \frac{25}{100}$ and $\frac{3}{4} = \frac{75}{100}$.

Once you have rewritten the fraction as its decimal equivalent, it is an easy process to convert the decimals into hundredths. The number written as 0.4 or $\frac{4}{10}$ is equivalent to 0.40 or $\frac{40}{100}$. Once you have hundredths, you have percent, since percent means "per hundred." 0.40 or $\frac{40}{100} = 40\%$. $\frac{1}{4} = \frac{25}{100} = 25\%$. $\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$ or 50%.

Multiplication by a Decimal is the Same as Division by a Whole Number

Notice that when you multiply a number by 10, the digit does not change, but it appears to move one place to the left, making the number larger. Consider the following examples:

Example 1

$7 \times 10 = 70$

Example 2

$28 \times 10 = 280$

Example 3

$0.09 \times 10 = 0.9$

When you multiply a number by 100, once again the digit(s) remain the same, but the number is much larger, and the digits appear to move two places to the left.

Example 4

$34 \times 100 = 3{,}400$

Example 5

$5.01 \times 100 = 501$

Example 6

$0.0006 \times 100 = 0.06$

Similarly, when dividing by 100 the digits appear to move two places to the right, because the result is much smaller.

Example 7

$451 \div 100 = 4.51$

Example 8

$2 \div 100 = 0.02$

Example 9

$7{,}809 \div 100 = 78.09$

Now let's apply this simple concept to decimals. A decimal is a fraction written on one line. When you multiply by a decimal numeral it is the same as multiplying by a fraction. The denominator of a fraction is another way of expressing division, since the line separating the numerator and denominator means "divided by."

Multiplying by 0.1 is the same as multiplying by $\frac{1}{10}$, which is another way of expressing "divide by 10." Similarly, multiplying by a decimal is the same as dividing by a whole number. Observe the following problems that illustrate different ways of expressing the same operation.

Example 10

$6 \times 0.1 = 0.6$

$6 \times \frac{1}{10} = \frac{6}{10} = 0.6$

$6 \div 10 = \frac{6}{10} = 0.6$

Example 11

$840 \times 0.01 = 8.4$

$840 \times \frac{1}{100} = \frac{840}{100} = 8.4$

$840 \div 100 = \frac{840}{100} = 8.4$

When you are asked to find a percentage of a number, simply rewrite the percentage as a decimal or a fraction and then multiply. Remember what we have just observed and don't be surprised that when we multiply by a decimal (that is less than 1), our answer will be much smaller than what we began with.

Example 12

Find 25% of 36.

Solution 12

$25\% = 0.25$ or $\frac{25}{100}$ and $0.25 \times 36 = 9$

So 25% of 36 is 9.

Example 13

Find 7% of 409.

Solution 13

$7\% = 0.07$ or $\frac{7}{100}$ and $0.07 \times 409 = 28.63$

So 7% of 409 = 28.63.

Working with Tenths

One tenth is the same as $\frac{1}{10}$ or 0.1 or $\frac{10}{100}$ or 10%. We have observed that one tenth means 1 divided by 10. When you divide by 10 or multiply by 0.1, move the decimal point over one place to the left. Another word for a tenth is a tithe.

Example 14

Find $\frac{1}{10}$ or 10% of 6.53.

Solution 14

$\frac{1}{10}$ or 10% or 0.1 of 6.53 is $0.1 \times 6.53 = 0.653$

Once you have mastered 10%, you can use this trick to figure out 20% or any multiple of 10%. For example, to find 20% of 4.13, first find 10% by moving the decimal point to the left, making it $\frac{1}{10}$ of what it was. Ten percent of 4.13 is 0.413. To find 20%, double that answer or multiply it by two, since 20% is two times 10%. The final answer is 0.826. We know that 20% is the same as $\frac{20}{100}$ or 0.20 written as a decimal. We could have multiplied 4.13 by 0.20 (or 0.2) and gotten 0.826, but I am trying to teach you to do this in your head!

Example 15

Find 20% of 175.

Solution 15

$\frac{1}{10}$ or 10% or 0.1 of 175 is $0.1 \times 175 = 17.50$

20% would be 2 times as much or 35.00.

Five percent is half of 10%. If you are asked to find 5% of a number, just find 10% and cut the answer in half. If you are buying a washing machine which costs $350.00, and there is a 5% tax, take 10% of the price, or $35.00 and cut it in half to get $17.50. Then add the tax on to the original price. So, $350 + $17.50 is $367.50, which is the real cost for the washer.

Sales Tax and Strategies for Computing Percentages

Almost every day we each pay sales tax. This tax varies from state to state. Including sales tax in your calculations, particularly when you are purchasing a large ticket item like a washing machine or a used car, helps you calculate the real cost of what you are buying.

In most of Pennsylvania we have a 6% sales tax. There are two ways to figure it. The most direct is to multiply the total by 0.06. Another way is to do it in your head. First find 5% by taking half of 10%. We still need to find 6%, which is 5% plus 1%. To find the 1%, move the decimal of the total amount two places to the left. If the amount you are being charged is $120.00, 10% is $12.00, and half of that is $6.00. One percent of $120.00 is $1.20, so the tax is $6.00 plus $1.20, or $7.20. The direct way is to multiply $120.00 by 0.06 to get $7.20. You could also multiply $120.00 by 6 and then move the decimal point two places to the left for $7.20. Either way will work.

Example 16

Adam purchased a new paddle for his kayak. It cost $48.00. What was the total bill if the sales tax is 5%?

Solution 16

This problem may be figured a few ways.

5% is half of 10% and 10% of $48.00 is $4.80, so 5% is $2.40

$48.00 + $2.40 = $50.40

Or, multiply $48.00 times 5%, which is $48.00 × 0.05 = $2.40

$48.00 + $2.40 = $50.40

Some cities and counties have a different sales tax than the state does, and the amount may be a mixed number like 8.25%.

Example 17

Teagan saw a nice sweatshirt at the mall. It cost $19.50.
What was the total bill if the sales tax is 8.25%?

Solution 17

$19.50 times 8.25% or $19.50 × 0.0825% = $1.60875 which rounds to $1.61.
The total is $19.50 + $1.61 = $21.11.

Tips

If you tip a waiter in an eating establishment, you have the opportunity to calculate percentages such as 15%. (The 15% is calculated on the amount of the food bill, not the food bill plus the tax.) I do this by figuring 10% and then taking half of that for the 5%. Here is an easy example. You get the bill and are ready to leave a tip. The bill is $24.00. So, 10% is $2.40 and half of that (5%) is $1.20. The $2.40 plus $1.20 is $3.60, which is the tip.

In real life, I like to round up or down depending on the service. If I got really good service, I would leave $4.00. If it was okay, but maybe not what it could be, a tip of $3.00 might do. I always leave something. Usually the waiters are paid a minimum salary plus their tips, so they do need to receive some tips. I have friends who will not leave any tip if the service was really bad. Everyone has to make their own decisions.

Depending on where you live, the standard tipping amount may vary. In most of the United States 15%–20% is considered standard. If you eat out for a special occasion with a number of guests, make sure you don't tip twice. Some restaurants automatically figure the tip into the bill when serving a large group.

In the past few years, the expected tip for servers in restaurants has moved from 15% to 20%. This is much easier to calculate. If the bill is $24.00, then 10% of 24.00 is $2.40, and you simply double that amount to arrive at $4.80 for the tip. The total for the food and a 20% tip is $28.80 ($24.00 + 4.80 = $28.80). Since I like to round up or down depending on the service, I would round up to $29.00 if it was good service. If the service was below average, I might make the final amount only $28.00, which includes a $4.00 tip. The lower tip is still over 15% (15% of $24.00 is $3.60).

Example 18

Darby's bill for lunch was $12.75. She had a wonderful server. The sales tax was 5%. How much was her tip and the final tally? How much could she add to make it a round number?

Solution 18

$12.75 × 20% = $2.55 for the tip
$12.75 × 5% = $0.64 for the tax
Total bill is $12.75 + $2.55 + $0.64 = $15.94
If you want it in even dollars, add $0.06 to make $16.00.

Example 19

Hannah took friends out to celebrate her first real estate sale. The price for breakfast was $46.25. The server was okay, but not special, and the sales tax was 6.5%. How much was the tip and the final tally? How much could be added to make it a round number?

Solution 19

$46.25 × 16% = $7.40 for the tip since the service was just okay.
$46.25 × 6.5% = $3.01 for the tax
Total bill is $46.25 + $7.40 + $3.01 = $56.66
If you want it in even dollars, add $0.34 to make $57.00.

Another Tip on Tipping

I have noticed that many eating establishments will now provide suggested tip amounts on the bottom of the check. The tip is calculated for 15%, 18%, and 20%. As I examined their computations, I discovered the percentage was figured on the price of the food PLUS the tax.

Example 20

The bill for the food was $40.00 and the sales tax was 8%. The total bill was $43.20. What is the difference between a tip computed on the price of the food alone and a tip computed with the tax included?

Solution 20

The percentages and resulting tip amounts on my bill were shown as:

15% $6.48	18% $7.78	20% $8.64

When I figured the tip based on the cost of food alone, the numbers were:

15% $6.00	18% $7.20	20% $8.00

At the Airport

When I am traveling, I carry several one-dollar bills with me. I tip the parking service if I leave a car at the garage because the driver helps me take my bags into his van and unloads them at the curb. I usually tip $1.00 per bag. If I use curbside boarding, I also tip the workers who help unload and check my bags. This service helps me avoid the line inside of the airport.

Example 21

Collin was flying to China and had three large bags with him. He first had to take a shuttle from the parking garage to the airport, and then have help checking his bags at the curbside baggage counter. How much did he spend in tips?

Solution 21

Collin tipped the shuttle driver $3.00 for his assistance. At the curbside baggage check-in, Collin gave the worker $5.00 for his help, since the bags were large.

Paying Bills

When you see “1% 10 Net 30 Days” on an invoice (an invoice is a bill for you to pay), it indicates two things. The “1% 10” means that if you pay your bill within 10 days, you can deduct 1% from the total. The “30” means that if you do not opt to make use of this discount, you still must pay within 30 days. Some companies will have a rate of 2% 15 Net 30 Days, which means that if you pay your bill within 15 days you save 2% of the total invoice.

Encouraging people to pay quickly is a positive thing for both those paying the invoice

and those being paid. If you as the consumer could pay 1% or 2% less on items you purchase, it would mean significant savings over a year. To the business offering the discount, cash flow, or having cash on hand with which to do business, is vital. They may have the best product and lots of sales, but if they aren't being paid in a timely fashion, they can't pay their suppliers and workers and eventually may have to close-up shop.

Example 22

Seth's printing bill for $785.00 was dated December 9. On the invoice it read "1% 10 net 30." It is now December 14th. How much should Seth pay?

Solution 22

Since it has been only five days since the date on the bill, Seth can deduct 1% or $7.85. The amount to pay is $777.15.

Example 23

Adam has a bill on his desk for $540.00. It reads "2% 15 net 30." It has been 17 days since the date on the bill. How much is he expected to send?

Solution 23

Adam missed the 15-day bonus period, so he has to pay the "net" or total amount of $540.00.

Property Taxes

Where I now live, we receive two tax bills for our home or property. One comes in the Spring and is called the Spring Tax. The larger one comes in the Fall and is referred to as the Fall Tax. Since these are large bills, there are several options for paying them. If you pay within the first two months of receiving the bill, you can deduct 5% from the total tax. If you pay a little later, you can save 2%, or you can wait a few months and pay it in full. If you wait too long, you will have to pay an additional fee of 1%.

Example 24

The Lloyd's property tax bill is $2,687.15 and was received on March 15.
They have the following options:
If paid in April or May receive a 5% discount.
If paid in June or July receive a 2% discount.
If paid in August or September pay full amount.
If paid after September 30 accrue a 1% penalty.

Option A: If they choose to pay their property tax bill on May 22, how much will it be?
Option B: What is the total amount if they send their payment on October 29?

Solution 24

Option A: Since it is in May, they can deduct 5%.
$2,687.15 × 0.05 = $134.36.
$2,687.15 - $134.36 = $2,552.79

Option B: If they are paying after September 30, they pay an additional 1%.
$2,687.15 × 0.01 = $26.87
$2,687.15 + $26.87 = $2,714.02

Rounding

When performing math with money, the standard practice is to not round off the numbers when working electronically. However, when a payment will be made by check or cash, you must round off at the hundredths place (cents), because that is the smallest unit of currency in the US. In the examples and solutions in this book, our general practice will be to not round off numbers in the middle of a calculation. Instead we will wait until we write the final answer before rounding to the nearest cent. In the rare case where the final answer has exactly five thousandths (and is therefore exactly between two numbers in the hundredths place) we will round up instead of down. If you end up with an answer that is within a few hundredths of the solution, it is probably only due to a difference in when and how you rounded the numbers.

Example 25

Divide $1.05 by 2 and then multiply by 3. Round the answer to the nearest cent.

Solution 25A (rounding at the end of the problem)

$1.05 ÷ 2 = $0.525
$0.525 × 3 = $1.575
Rounding the 5 in the thousandths place makes the final answer $1.58

Solution 25B (rounding in the middle of the problem)

$1.05 ÷ 2 = $0.525 which rounds to $0.053
$0.53 × 3 = $1.59